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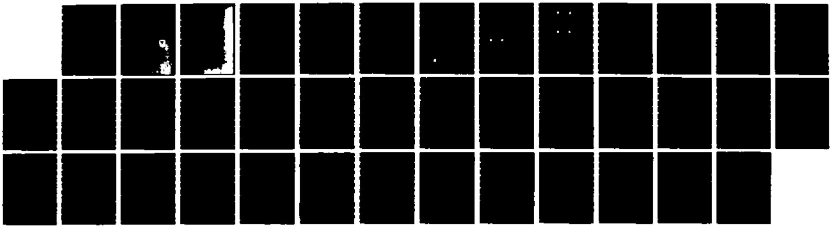

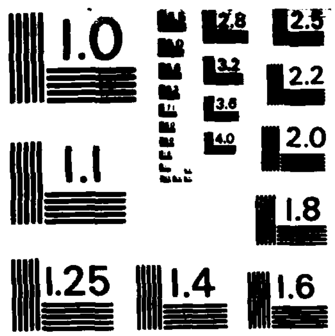

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On the Form of the Collective Bremsstrahlung Recoil Force in a Nonequilibrium Relativistic Beam-Plasma System

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

> The general form is derived, from first principles, for the collective bremsstrahlung recoil force on a test particle participating in a bremsstrahlung process in a relativistic nonequilibrium beam-plasma system. The force is expressed in terms of the particle and photon distribution functions and the bremsstrahlung transition rate. This relationship is needed in calculations of collective radiation processes and the conditions for the occurrence of bremsstrahlung radiative instability in relativistic beam-plasma systems.

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FIGURES

1. INTRODUCTION

The collective bremsstrahlung recoil force on a relativistic test particle participating in a bremsstrahlung process in a nonequilibrium relativistic beam-plasma system is given by[1,2]

$$\vec{F}_{\alpha}^{\sigma} = -\int (2\pi)^{-9}\, d^3\vec{p}_{\beta}\, d^3\vec{\kappa}\, d^3\vec{k}\; \vec{k}(\vec{\kappa} - \vec{k})\cdot\left(\vec{\nabla}_{p_\beta} f_{\vec{p}_\beta}\right)$$
$$\times\, v^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k}) N^{\sigma}_{\vec{k}}\, \delta\left(\omega^{\sigma}_{k} - \vec{\kappa}\cdot\vec{v}_{\alpha} + (\vec{\kappa} - \vec{k})\cdot\vec{v}_{\beta}\right) \quad . \tag{1}$$

Here, $v^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k})\delta\left(\omega^{\sigma}_{k} - \vec{\kappa}\cdot\vec{v}_{\alpha} + (\vec{\kappa} - \vec{k})\cdot\vec{v}_{\beta}\right)$ is the bremsstrahlung transition rate (probability per unit time) for scattering of particles of species α and β, velocities v_α and v_β, and momenta p_α and p_β, where the scattering produces momentum transfer $\vec{\kappa}$ and the emission of a photon in mode σ with wave vector $\vec{k}$ and frequency $\omega^{\sigma}_{\vec{k}}$; $N^{\sigma}_{\vec{k}}$ is the photon (plasmon) distribution function; and $f_{\vec{p}_\beta}$ is the charged particle distribution function. Here and throughout, the units are chosen such that $\hbar = 1$. In this report equation (1) is derived from first principles. This expression is useful in obtaining an expression for the collective bremsstrahlung transition rate by direct comparison with another expression for the collective bremsstrahlung recoil force, which is determined from the equation of motion for a dynamically polarized test particle undergoing bremsstrahlung.[1,2] This transition rate is important in calculations of collective bremsstrahlung and the conditions for the occurrence of the bremsstrahlung radiative instability in relativistic beam-plasma systems (see the work of Akopyan and Tsytovich, Selected Bibliography).

In section 2 the particle balance equations, including bremsstrahlung and inverse bremsstrahlung processes, are derived in terms of the basic bremsstrahlung transition rate. In section 3, the soft photon or quasiclassical limit of these equations is obtained and separated into spontaneous and

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[1] *V. N. Tsytovich, Bremsstrahlung of a Relativistic Plasma, Tr. Fiz. Inst. Akad. Nauk SSSR, 66 (1973), 191 [Proc (TRUDY) of P. N. Lebedev Physics Inst., 66 (1975), 199].*

[2] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plasmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].*

induced contributions. Expressions are obtained for the diffusion coefficient and induced recoil force due to induced bremsstrahlung. In section 4, the general form of the induced recoil force is determined in terms of the bremsstrahlung radiation field, particle distributions, dielectric permittivity, and bremsstrahlung transition rate. Section 5 summarizes the main results of the derivation.

2. PARTICLE BALANCE EQUATIONS

We here derive the particle balance equations in terms of the basic bremsstrahlung transition rate, for beam-plasma configurations in which bremsstrahlung and inverse bremsstrahlung are the dominant processes. For this purpose, consider the elementary bremsstrahlung process depicted in figure 1. Here a particle of species α and momentum $\vec{p}_\alpha$ scatters off a particle of species β and momentum $\vec{p}_\beta$ and emits a photon of momentum $\vec{k}$. Figure 1 is not a Feynman diagram since it represents the probability for the process and not its amplitude. If the momentum decrease of particle α is denoted by $\vec{\kappa}$, then the momentum $\vec{p}\,'_\alpha$ of particle α after the collision is given by

$$\vec{p}\,'_\alpha = \vec{p}_\alpha - \vec{\kappa} \quad . \tag{2}$$

Then, by conservation of momentum, the momentum of particle β after the collision is given by

$$\vec{p}\,'_\beta = \vec{p}_\beta + \vec{\kappa} - \vec{k} \quad . \tag{3}$$

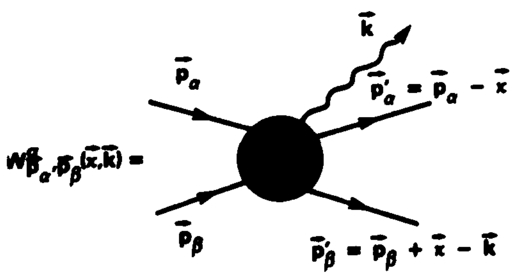


Figure 1. Elementary bremsstrahlung process which removes particles of species α from point $\vec{p}_\alpha$ in momentum space.

The quantity $w^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k})$ is the probability per unit time that in the scattering of a particle of the species α and initial momentum $\vec{p}_\alpha$ off a particle of species β and initial momentum $\vec{p}_\beta$, a photon of momentum $\vec{k}$ in mode σ is emitted, and there occurs momentum transfer $\vec{\kappa} - \vec{k}$ to particle β. By time-reversal invariance, the transition rate for the inverse process (in which a particle of momentum $\vec{p}_\alpha$ absorbs a photon of momentum $\vec{k}$ and scatters off a particle of momentum $\vec{p}_\beta$ with momentum transfer $\vec{\kappa}$ to particle α) is then given by $w^{\sigma}_{\vec{p}_\alpha+\vec{\kappa},\vec{p}_\beta-\vec{\kappa}+\vec{k}}(\vec{\kappa},\vec{k})$. This process is depicted in figure 2 in the leftmost figure, which is the time-reversed process of the figure to its right; therefore, the transition probabilities of the two processes are equal. The inverse process is denoted by $w^{\sigma}_{\vec{p}_\alpha+\vec{\kappa},\vec{p}_\beta-\vec{\kappa}+\vec{k}}(\vec{\kappa},\vec{k})$ in the notation of figure 1. The equality of the probabilities of the direct and inverse processes due to time-reversal invariance is also known as the principle of microscopic reversibility or reciprocity, and leads to the principle of detailed balance. The processes of figures 1 and 2 both deplete the density of particles of type α at point $\vec{p}_\alpha$ in momentum space by adding or subtracting momentum.

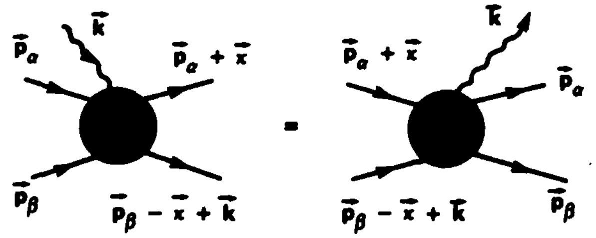
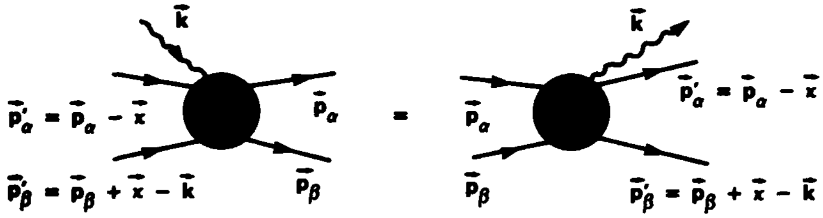


Figure 2. Inverse bremsstrahlung process which removes particles of species α from point $\vec{p}_\alpha$ in momentum space. First equality follows from time-reversal invariance.

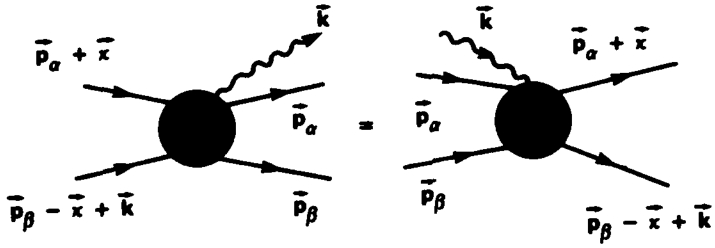
$$= W^{\sigma}_{\vec{p}_\alpha+\vec{\kappa},\vec{p}_\beta-\vec{\kappa}+\vec{k}}(\vec{\kappa},\vec{k})$$

The particle density at point $\vec{p}_\alpha$ increases because of the following two processes. The inverse process to that in figure 1 produces gain at point $\vec{p}_\alpha$. By time-reversal invariance, this is again given by $w^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k})$, as depicted in figure 3. Similarly, the time reversal of the process in figure 2 also adds particles at point $\vec{p}_\alpha$ with transition rate $w^{\sigma}_{\vec{p}_\alpha+\vec{\kappa},\vec{p}_\beta-\vec{\kappa}+\vec{k}}(\vec{\kappa},\vec{k})$, as depicted in figure 4.



$= W_{\vec{p}_\alpha, \vec{p}_\beta}^{\beta}(\vec{x},\vec{k})$

Figure 3. This process adds particles at point $\vec{p}_\alpha$ in a momentum space.



$= W_{\vec{p}_\alpha + \vec{x}, \vec{p}_\beta - \vec{x} + \vec{k}}^{\beta}(\vec{x},\vec{k})$

Figure 4. This process also adds particles at point $\vec{p}_\alpha$ in momentum space.

Taking these four processes into account including both spontaneous and induced emission, we find that the particle-balance equation giving the time rate of change of the particle distribution $f_{\vec{p}\alpha}$ for particles of type α at point $\vec{p}_\alpha$ in momentum space is

$$\frac{\partial f_{\vec{p}_\alpha}}{\partial t} = -\int (2\pi)^{-9}\, d^3\vec{k}\, d^3\vec{p}_\beta\, d^3\vec{\kappa}\, f_{\vec{p}_\alpha} f_{\vec{p}_\beta} W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k}) \left(N^{\sigma}_{\vec{k}} + 1\right)$$

$$- \int (2\pi)^{-9}\, d^3\vec{k}\, d^3\vec{p}_\beta\, d^3\vec{\kappa}\, N^{\sigma}_{\vec{k}} f_{\vec{p}_\alpha} f_{\vec{p}_\beta} W^{\sigma}_{\vec{p}_\alpha+\vec{\kappa},\vec{p}_\beta-\vec{\kappa}+\vec{k}}(\vec{\kappa},\vec{k})$$

$$+ \int (2\pi)^{-9}\, d^3\vec{k}\, d^3\vec{p}_\beta\, d^3\vec{\kappa}\, N^{\sigma}_{\vec{k}}\, f_{\vec{p}_\alpha-\vec{\kappa}}\, f_{\vec{p}_\beta+\vec{\kappa}-\vec{k}}\, W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k}) \tag{4}$$

$$+ \int (2\pi)^{-9}\, d^3\vec{k}\, d^3\vec{p}_\beta\, d^3\vec{\kappa}\, f_{\vec{p}_\alpha+\vec{\kappa}}\, f_{\vec{p}_\beta-\vec{\kappa}+\vec{k}}\, W^{\sigma}_{\vec{p}_\alpha+\vec{\kappa},\vec{p}_\beta-\vec{\kappa}+\vec{k}}(\vec{\kappa},\vec{k})\left(N^{\sigma}_{\vec{k}} + 1\right) .$$

The first term in equation (4) represents the rate of decrease of particle density at point $\vec{p}_\alpha$ due to the bremsstrahlung process of figure 1. The function $f_{\vec{p}_\beta}$ is the distribution function for particles of species β, and $N^{\sigma}_{\vec{k}}$ is the photon distribution function. The factor $N^{\sigma}_{\vec{k}} + 1$ takes account of both induced and spontaneous emission. An integral over all possible photon wave vectors, scattered particle momenta $\vec{p}_\beta$, and momentum transfers $\vec{\kappa}$ also necessarily appears in equation (4). The factors of 2π arise from counting quantum states. Thus, for example, $d^3\vec{k}/(2\pi)^3$ is the number of quantum states per unit volume with momentum $\hbar\vec{k}$ in the interval $d^3\vec{k}$. The same phase-space normalization for the particle distribution function as in Tsytovich[1] is used here, namely,

$$n_\alpha = \int \frac{d^3\vec{p}_\alpha}{(2\pi)^3} f_{\vec{p}_\alpha} \quad , \tag{5}$$

where n_α is the number of particles of species α per unit volume. The second term in equation (4) represents the rate of decrease due to the inverse bremsstrahlung process of figure 2. The third term represents the rate of increase

[1]*V. N. Tsytovich, Bremsstrahlung of a Relativistic Plasma, Tr. Fiz. Inst. Akad. Nauk SSSR,* <u>66</u> *(1973), 191 [Proc (TRUDY) of P. N. Lebedev Physics Inst.,* <u>66</u> *(1975), 199].*

due to the inverse bremsstrahlung process of figure 3. The fourth term represents the rate of increase due to the bremsstrahlung process of figure 4.

Combining terms in equation (4) produces

$$\frac{\partial f_{\vec{p}_\alpha}}{\partial t} = -\int \frac{d^3\vec{k}\, d^3\vec{p}_\beta\, d^3\vec{\kappa}}{(2\pi)^9} W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k}) \left[f_{\vec{p}_\alpha} f_{\vec{p}_\beta} \left(N^{\sigma}_{\vec{k}} + 1\right) - N^{\sigma}_{\vec{k}} f_{\vec{p}_\alpha-\vec{\kappa}} f_{\vec{p}_\beta+\vec{\kappa}-\vec{k}} \right]$$
$$- \int \frac{d^3\vec{k}\, d^3\vec{p}_\beta\, d^3\vec{\kappa}}{(2\pi)^9} W^{\sigma}_{\vec{p}_\alpha+\vec{\kappa},\vec{p}_\beta-\vec{\kappa}+\vec{k}}(\vec{\kappa},\vec{k}) \left[f_{\vec{p}_\alpha} f_{\vec{p}_\beta} N^{\sigma}_{\vec{k}} - f_{\vec{p}_\alpha+\vec{\kappa}} f_{\vec{p}_\beta-\vec{\kappa}+\vec{k}} \left(N^{\sigma}_{\vec{k}} + 1\right) \right] . \tag{6}$$

Breaking equation (6) into its induced and spontaneous parts results in

$$\frac{\partial f_{\vec{p}_\alpha}}{\partial t} = \left(\frac{\partial f_{\vec{p}_\alpha}}{\partial t}\right)_i + \left(\frac{\partial f_{\vec{p}_\alpha}}{\partial t}\right)_s \tag{7}$$

where the induced part $\left(\partial f_{\vec{p}_\alpha}/\partial t\right)_i$ is that proportional to the photon number density $N^{\sigma}_{\vec{k}}$ and the spontaneous part $\left(\partial f_{\vec{p}_\alpha}/\partial t\right)_s$ is that independent of $N^{\sigma}_{\vec{k}}$. Thus

$$\left(\frac{\partial f_{\vec{p}_\alpha}}{\partial t}\right)_i = -\int \frac{d^3\vec{k}\, d^3\vec{p}_\beta\, d^3\vec{\kappa}}{(2\pi)^9} N^{\sigma}_{\vec{k}} \left\{ W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k}) \left[f_{\vec{p}_\alpha} f_{\vec{p}_\beta} - f_{\vec{p}_\alpha-\vec{\kappa}} f_{\vec{p}_\beta+\vec{\kappa}-\vec{k}} \right] \right. \tag{8}$$
$$\left. + W^{\sigma}_{\vec{p}_\alpha+\vec{\kappa},\vec{p}_\beta-\vec{\kappa}+\vec{k}}(\vec{\kappa},\vec{k}) \left[f_{\vec{p}_\alpha} f_{\vec{p}_\beta} - f_{\vec{p}_\alpha+\vec{\kappa}} f_{\vec{p}_\beta-\vec{\kappa}+\vec{k}} \right] \right\}$$

$$\left(\frac{\partial f_{\vec{p}_\alpha}}{\partial t}\right)_s = -\int \frac{d^3\vec{k}\, d^3\vec{p}_\beta\, d^3\vec{\kappa}}{(2\pi)^9} \Big[W^\sigma_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k}) f_{\vec{p}_\alpha} f_{\vec{p}_\beta} \tag{9}$$

$$- W^\sigma_{\vec{p}_\alpha+\vec{k},\vec{p}_\beta-\vec{k}+\vec{\kappa}}(\vec{\kappa},\vec{k}) f_{\vec{p}_\alpha+\vec{k}}\, f_{\vec{p}_\beta-\vec{k}+\vec{\kappa}} \Big] .$$

3. SOFT PHOTON AND SMALL MOMENTUM TRANSFER APPROXIMATION

We next assume that the momentum transfer $\vec{\kappa}$ and the momentum $\vec{k}$ of the radiated quantum are small relative to the relativistic particle momenta $\vec{p}_\alpha$ and $\vec{p}_\beta$, thereby enabling Taylor series expansions for $W^\sigma_{\vec{p}_\alpha+\vec{k},\vec{p}_\beta-\vec{\kappa}+\vec{k}}$, $f_{\vec{p}_\alpha-\vec{\kappa}}$, $f_{\vec{p}_\beta+\vec{\kappa}-\vec{k}}$, $f_{\vec{p}_\alpha+\vec{\kappa}}$, and $f_{\vec{p}_\beta-\vec{k}+\vec{\kappa}}$ in equation (8). Thus we assume that

$$\{|\vec{\kappa}|,|\vec{k}|\} \ll \{|\vec{p}_\alpha|,|\vec{p}_\beta|\} . \tag{10}$$

This is evidently consistent with the Born approximation for plasma, namely that the relativistic particle momentum is much greater than the electromagnetic impulse received by a plasma particle in a time interval given by the inverse plasma frequency.[1-5] First expanding $W^\sigma_{\vec{p}_\alpha+\vec{k},\vec{p}_\beta-\vec{k}+\vec{\kappa}}$ about $\vec{p}_\alpha$ in the first variable, we obtain

[1] *V. N. Tsytovich, Bremsstrahlung of a Relativistic Plasma, Tr. Fiz. Inst. Akad. Nauk SSSR, 66 (1973), 191 [Proc (TRUDY) of P. N. Lebedev Physics Inst., 66 (1975), 199].*

[2] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plasmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975) 371].*

[3] *H. E. Brandt, Nonlinear Dynamic Polarization Force on a Relativistic Test Particle in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1994 (1983).*

[4] *H. E. Brandt, Nonlinear Force on an Unpolarized Relativistic Test Particle to Second-Order in the Total Field in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1995 (1983).*

[5] *H. E. Brandt, The Total Field in Collective Bremsstrahlung in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1996 (1983).*

$$W^{\sigma}_{\vec{p}_\alpha+\vec{\kappa},\vec{p}_\beta-\vec{\kappa}+\vec{k}} = W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta-\vec{\kappa}+\vec{k}} + \kappa_i \frac{\partial}{\partial p_{\alpha i}} W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta-\vec{\kappa}+\vec{k}} + \frac{1}{2}\kappa_i\kappa_j \frac{\partial^2 W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta-\vec{\kappa}+\vec{k}}}{\partial p_{\alpha i}\partial p_{\alpha j}} \quad . \tag{11}$$

Next expanding equation (11) to second order about $\vec{p}_\beta$ in the second variable gives

$$W^{\sigma}_{\vec{p}_\alpha+\vec{\kappa},\vec{p}_\beta-\vec{\kappa}+\vec{k}} = W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta} + (k_i - \kappa_i)\frac{\partial}{\partial p_{\beta i}} W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta} + \frac{1}{2}(k_i - \kappa_i)(k_j - \kappa_j)\frac{\partial^2 W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}}{\partial p_{\beta i}\partial p_{\beta j}} + \kappa_i \frac{\partial W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}}{\partial p_{\alpha i}} + \kappa_i(k_j - \kappa_j)\frac{\partial^2 W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}}{\partial p_{\alpha i}\partial p_{\beta j}} + \frac{1}{2}\kappa_i\kappa_j \frac{\partial^2 W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}}{\partial p_{\alpha i}\partial p_{\alpha j}} \quad . \tag{12}$$

Also in equation (8) the following Taylor series expansion applies:

$$\left[f_{\vec{p}_\alpha} f_{\vec{p}_\beta} - f_{\vec{p}_\alpha-\vec{\kappa}}\, f_{\vec{p}_\beta+\vec{\kappa}-\vec{k}}\right] = f_{\vec{p}_\alpha} f_{\vec{p}_\beta} - \left[f_{\vec{p}_\alpha} - \kappa_i \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha i}} + \frac{1}{2}\kappa_i\kappa_j \frac{\partial^2 f_{\vec{p}_\alpha}}{\partial p_{\alpha i}\partial p_{\alpha j}}\right]\left[f_{\vec{p}_\beta} + (\kappa_\ell - k_\ell)\frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta \ell}} + \frac{1}{2}(\kappa_\ell - k_\ell)(\kappa_m - k_m)\frac{\partial^2 f_{\vec{p}_\beta}}{\partial p_{\beta \ell}\partial p_{\beta m}}\right] \; ; \tag{13}$$

or combining terms produces

$$\begin{aligned}\left[f_{\vec{p}_\alpha} f_{\vec{p}_\beta} - f_{\vec{p}_\alpha - \vec{\kappa}}\, f_{\vec{p}_\beta + \vec{\kappa} - \vec{k}}\right] = {} & -(\kappa_\ell - k_\ell) f_{\vec{p}_\alpha} \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta\ell}} \\ & - \frac{1}{2} (\kappa_\ell - k_\ell)(\kappa_m - k_m) f_{\vec{p}_\alpha} \frac{\partial^2 f_{\vec{p}_\beta}}{\partial p_{\beta\ell} \partial p_{\beta m}} + \kappa_i \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha i}} f_{\vec{p}_\beta} \\ & + \kappa_i (\kappa_\ell - k_\ell) \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha i}} \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta\ell}} - \frac{1}{2} \kappa_i \kappa_j \frac{\partial^2 f_{\vec{p}_\alpha}}{\partial p_{\alpha i} \partial p_{\alpha j}} f_{\vec{p}_\beta} \quad . \end{aligned} \tag{14}$$

Also,

$$\begin{aligned}\left[f_{\vec{p}_\alpha} f_{\vec{p}_\beta} - f_{\vec{p}_\alpha + \vec{\kappa}}\, f_{\vec{p}_\beta - \vec{\kappa} + \vec{k}}\right] = {} & -(k_i - \kappa_i) f_{\vec{p}_\alpha} \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta i}} \\ & - \frac{1}{2} (k_\ell - \kappa_\ell)(k_m - \kappa_m) f_{\vec{p}_\alpha} \frac{\partial^2 f_{\vec{p}_\beta}}{\partial p_{\beta\ell} \partial p_{\beta m}} - \kappa_i f_{\vec{p}_\beta} \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha i}} \\ & - \kappa_i (k_\ell - \kappa_\ell) \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha i}} \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta\ell}} - \frac{1}{2} \kappa_i \kappa_j f_{\vec{p}_\beta} \frac{\partial^2 f_{\vec{p}_\alpha}}{\partial p_{\alpha i} \partial p_{\alpha j}} \quad . \end{aligned} \tag{15}$$

Substituting equations (12), (14), and (15) in equation (8) produces

$$\left(\frac{\partial f_{p_\alpha}}{\partial t}\right)_i = -\int \frac{d^3\vec{k}\ d^3\vec{p}_\beta\ d^3\vec{\kappa}}{(2\pi)^9} N^\sigma_{\vec{k}} \Bigg\{ W^\sigma_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k}) \Bigg[-(\kappa_\ell - k_\ell) f_{\vec{p}_\alpha} \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta\ell}}$$

$$- \frac{1}{2}(\kappa_\ell - k_\ell)(\kappa_m - k_m) f_{\vec{p}_\alpha} \frac{\partial^2 f_{\vec{p}_\beta}}{\partial p_{\beta\ell}\partial p_{\beta m}} + \kappa_i \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha i}} f_{\vec{p}_\beta}$$

$$+ \kappa_i(\kappa_\ell - k_\ell) \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha i}} \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta\ell}} - \frac{1}{2}\kappa_i\kappa_j \frac{\partial^2 f_{\vec{p}_\alpha}}{\partial p_{\alpha i}\partial p_{\alpha j}} f_{\vec{p}_\beta}$$

$$- (k_i - \kappa_i) f_{\vec{p}_\alpha} \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta i}} - \frac{1}{2}(k_\ell - \kappa_\ell)(k_m - \kappa_m) f_{\vec{p}_\alpha} \frac{\partial^2 f_{\vec{p}_\beta}}{\partial p_{\beta\ell}\partial p_{\beta m}} - \kappa_i f_{\vec{p}_\beta} \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha i}}$$

$$- \kappa_i(k_\ell - \kappa_\ell) \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha i}} \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta\ell}} - \frac{1}{2}\kappa_i\kappa_j f_{\vec{p}_\beta} \frac{\partial^2 f_{\vec{p}_\alpha}}{\partial p_{\alpha i}\partial p_{\alpha j}} \Bigg] \tag{16}$$

$$+ \frac{\partial W^\sigma_{\vec{p}_\alpha,\vec{p}_\beta}}{\partial p_{\beta i}} \left[-(k_i - \kappa_i)(k_m - \kappa_m) f_{\vec{p}_\alpha} \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta m}} - (k_i - \kappa_i)\kappa_m f_{\vec{p}_\beta} \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha m}} \right]$$

$$+ \frac{\partial W^\sigma_{\vec{p}_\alpha,\vec{p}_\beta}}{\partial p_{\alpha i}} \left[-\kappa_i(k_m - \kappa_m) f_{\vec{p}_\alpha} \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta m}} - \kappa_i\kappa_\ell f_{\vec{p}_\beta} \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha\ell}} \right] \Bigg\} .$$

The first and sixth terms of equation (16) cancel. Also, the third and eighth terms cancel. Combining the fifth and tenth terms, the second and seventh terms, and the fourth and ninth terms, and integrating the eleventh and twelfth terms by parts, we find that equation (16) becomes

$$\left(\frac{\partial f_{\vec{p}_\alpha}}{\partial t}\right)_i = -\int \frac{d^3\vec{k}\; d^3\vec{p}_\beta\; d^3\vec{\kappa}}{(2\pi)^9} N^{\sigma}_{\vec{k}} \Bigg\{ W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k}) \Bigg[-(\kappa_\ell - k_\ell)(\kappa_m - k_m) f_{\vec{p}_\alpha} \frac{\partial^2 f_{\vec{p}_\beta}}{\partial p_{\beta\ell}\partial p_{\beta m}}$$

$$+ 2\kappa_i(\kappa_\ell - k_\ell) \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha i}} \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta \ell}} - \kappa_i\kappa_j \frac{\partial^2 f_{\vec{p}_\alpha}}{\partial p_{\alpha i}\partial p_{\alpha j}} f_{\vec{p}_\beta} \Bigg]$$

$$+ W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta} \Bigg[(k_i - \kappa_i)(k_m - \kappa_m) f_{\vec{p}_\alpha} \frac{\partial^2 f_{\vec{p}_\beta}}{\partial p_{\beta m}\partial p_{\beta i}} \tag{17}$$

$$+ (k_i - \kappa_i)\kappa_m \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta i}} \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha m}} \Bigg]$$

$$+ \frac{\partial W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}}{\partial p_{\alpha i}} \Bigg[-\kappa_i(k_m - \kappa_m) f_{\vec{p}_\alpha} \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta m}} - \kappa_i\kappa_\ell f_{\vec{p}_\beta} \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha \ell}} \Bigg] \Bigg\} .$$

The first and fourth terms cancel; the second and fifth terms combine; and after terms are rearranged, equation (17) becomes

$$\left(\frac{\partial f_{\vec{p}_\alpha}}{\partial t}\right)_i = \int \frac{d^3\vec{k}\; d^3\vec{p}_\beta\; d^3\vec{\kappa}}{(2\pi)^9} \Bigg[\kappa_i\kappa_j \frac{\partial W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}}{\partial p_{\alpha i}} N^{\sigma}_{\vec{k}} f_{\vec{p}_\beta} \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha j}}$$

$$+ \kappa_i\kappa_j W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k}) N^{\sigma}_{\vec{k}} f_{\vec{p}_\beta} \frac{\partial^2 f_{\vec{p}_\alpha}}{\partial p_{\alpha i}\partial p_{\alpha j}}$$

$$- \kappa_i(\kappa_j - k_j) \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta j}} \frac{\partial W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k})}{\partial p_{\alpha i}} N^{\sigma}_{\vec{k}} f_{\vec{p}_\alpha} \tag{18}$$

$$- \kappa_i(\kappa_j - k_j) \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta j}} W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k}) N^{\sigma}_{\vec{k}} \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha i}} \Bigg] .$$

Equivalently,

$$\left(\frac{\partial f_{\vec{p}_\alpha}}{\partial t}\right)_i = \frac{\partial}{\partial p_{\alpha i}}\left\{\left[\int \frac{d^3\vec{k}\ d^3\vec{\kappa}\ d^3\vec{p}_\beta}{(2\pi)^9}\ \kappa_i\kappa_j W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\kappa,k) N^{\sigma}_{\vec{k}}\ f_{\vec{p}_\beta}\right]\frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha_j}}\right\}$$

$$+ \frac{\partial}{\partial p_{\alpha i}}\left\{\left[-\int \frac{d^3\vec{k}\ d^3\vec{\kappa}\ d^3\vec{p}_\beta}{(2\pi)^9}\ \kappa_i(\kappa_j - k_j)\ \frac{\partial f_{\vec{p}_\beta}}{\partial p_{\beta j}}\ W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k}) N^{\sigma}_{\vec{k}}\right] f_{p_\alpha}\right\} . \tag{19}$$

4. INDUCED RECOIL FORCE DUE TO BREMSSTRAHLUNG

Equation (19) may be rewritten in the form of a diffusion equation as follows:

$$\left(\frac{\partial f_{\vec{p}_\alpha}}{\partial t}\right)_i = \frac{\partial}{\partial p_{\alpha i}}\left(D^{\sigma}_{\alpha ij}\ \frac{\partial f_{\vec{p}_\alpha}}{\partial p_{\alpha j}}\right) + \frac{\partial}{\partial p_{\alpha i}}\left(F^{\sigma}_{\alpha i}\ f_{\vec{p}_\alpha}\right) , \tag{20}$$

where the induced diffusion coefficient $D^{\sigma}_{\alpha ij}$ is given by

$$D^{\sigma}_{\alpha ij} = \int \frac{d^3\vec{k}\ d^3\vec{\kappa}}{(2\pi)^6}\ \frac{d^3\vec{p}_\beta}{(2\pi)^3}\ \kappa_i\kappa_j W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k}) N^{\sigma}_{\vec{k}}\ f_{\vec{p}_\beta} , \tag{21}$$

and the induced dynamic friction force or bremsstrahlung recoil force is given by

$$\vec{F}^{\sigma}_{\alpha} = -\int \frac{d^3\vec{k}\ d^3\vec{\kappa}\ d^3\vec{p}_\beta}{(2\pi)^9}\ \vec{\kappa}(\vec{\kappa} - \vec{k})\cdot(\vec{\nabla}_{p_\beta} f_{\vec{p}_\beta}) W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k}) N^{\sigma}_{\vec{k}} . \tag{22}$$

Equation (22) is the sought-after expression for the recoil force on particle α due to the induced bremsstrahlung process. Energy conservation can be explicitly factored out of the transition rate appearing in equation (22) as follows. The bremsstrahlung transition probability $W^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k})$ in equation (22) must conserve energy. Therefore

$$\varepsilon_{\vec{p}_\alpha} + \varepsilon_{\vec{p}_\beta} = \varepsilon_{\vec{p}_\alpha - \vec{\kappa}} + \varepsilon_{\vec{p}_\beta + \vec{\kappa} - \vec{k}} + \omega_k^\sigma \quad , \tag{23}$$

where $\varepsilon_{\vec{p}_\alpha}$ denotes the total energy of the particle of species α and momentum $\vec{p}_\alpha$. Using the conditions in equation (10) produces, to lowest order,

$$\varepsilon_{\vec{p}_\alpha - \vec{\kappa}} = \varepsilon_{\vec{p}_\alpha} - \vec{\kappa}\cdot\vec{\nabla}_{p_\alpha}\varepsilon_{\vec{p}_\alpha} \tag{24}$$

and

$$\varepsilon_{\vec{p}_\beta + \vec{\kappa} - \vec{k}} = \varepsilon_{\vec{p}_\beta} + \left(\vec{\kappa} - \vec{k}\right)\cdot\vec{\nabla}_{p_\beta}\varepsilon_{\vec{p}_\beta} \quad . \tag{25}$$

Next, using relativistic kinematics, we obtain

$$\vec{\nabla}_{p_\alpha}\varepsilon_{\vec{p}_\alpha} = \vec{\nabla}_{p_\alpha}\left[(m_\alpha c^2)^2 + p_\alpha^2 c^2\right]^{1/2} = \frac{\vec{p}_\alpha c^2}{\varepsilon_{\vec{p}_\alpha}} = \vec{v}_\alpha \quad . \tag{26}$$

Using equation (26) in equations (23) to (25), then to lowest order in $\vec{k}$ and $\vec{\kappa}$, we obtain

$$\omega_k^\sigma = \vec{\kappa}\cdot\vec{v}_\alpha + \left(\vec{k} - \vec{\kappa}\right)\cdot\vec{v}_\beta \quad . \tag{27}$$

Factoring this expression of energy conservation explicitly into the bremsstrahlung probability $w^\sigma_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k})$, we define the quantity $v^\sigma_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k})$ by

$$w^\sigma_{\vec{p}_\alpha,\vec{p}_\beta}\left(\vec{\kappa},\vec{k}\right) = v^\sigma_{\vec{p}_\alpha,\vec{p}_\beta}\left(\vec{\kappa},\vec{k}\right)\delta\left(\omega_k^\sigma - \vec{\kappa}\cdot\vec{v}_\alpha + \left(\vec{\kappa} - \vec{k}\right)\cdot\vec{v}_\beta\right) \quad . \tag{28}$$

Substituting equation (28) in equation (22) gives

$$\vec{F}_{\alpha}^{\sigma} = - \int \frac{d^3\vec{p}_\beta \, d^3\vec{\kappa} \, d^3\vec{k}}{(2\pi)^9} \vec{\kappa}(\vec{\kappa} - \vec{k})\cdot(\vec{\nabla}_{p_\beta} f_{\vec{p}_\beta}) v^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k})$$

$$\times N^{\sigma}_{\vec{k}} \delta\left(\omega^{\sigma}_{\vec{k}} - \vec{k}\cdot\vec{v}_\alpha + (\vec{k} - \vec{\kappa})\cdot\vec{v}_\beta\right) \quad . \tag{29}$$

Equation (29) expresses the recoil force on a particle of species α due to induced bremsstrahlung as an integral over particle momentum $\vec{p}_\beta$, photon wave vector $\vec{k}$, and momentum transfer $\vec{\kappa}$. The integrand involves the particle distribution $f_{\vec{p}_\beta}$ of the scattering particle, the bremsstrahlung transition rate $v^{\sigma}_{\vec{p}_\alpha,\vec{p}_\beta}(\vec{\kappa},\vec{k})$, and the photon distribution $N^{\sigma}_{\vec{k}}$.

The delta function in equation (29) expresses energy conservation in the bremsstrahlung process in the soft photon and small momentum transfer approximation for relativistic particles. Equation (29) agrees with Tsytovich's (4.1)[1] exactly. It differs in sign from equation (3) of Akopyan and Tsytovich,[2] which apparently has a typographical error.

The photon density $N^{\sigma}_{\vec{k}}$ in equation (29) can be expressed in terms of the associated field as follows. The power density delivered to the electromagnetic field is given by

$$\frac{du}{dt} = -\vec{j}\cdot\vec{E} \quad . \tag{30}$$

[1]*V. N. Tsytovich, Bremsstrahlung of a Relativistic Plasma, Tr. Fiz. Inst. Akad. Nauk SSSR, 66 (1973), 191 [Proc (TRUDY) of P. N. Lebedev Physics Inst., 66 (1975), 199].*

[2]*A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plasmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].*

Using Maxwell's equation

$$\vec{\nabla}\times\vec{H} = \vec{j} + \frac{\partial\vec{D}}{\partial t} \qquad (31)$$

in equation (30) produces

$$\frac{du}{dt} = -\left(\vec{\nabla}\times\vec{H} - \frac{\partial\vec{D}}{\partial t}\right)\cdot\vec{E} \quad . \qquad (32)$$

Equivalently,

$$\frac{du}{dt} = \vec{\nabla}\cdot\left(\vec{E}\times\vec{H}\right) - \vec{H}\cdot\vec{\nabla}\times\vec{E} + \vec{E}\cdot \frac{\partial\vec{D}}{\partial t} \quad . \qquad (33)$$

Substituting Maxwell's equation

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad (34)$$

in equation (33) produces

$$\frac{du}{dt} = \vec{\nabla}\cdot\left(\vec{E}\times\vec{H}\right) + \left(\vec{H}\cdot \frac{\partial\vec{B}}{\partial t} + \vec{E}\cdot \frac{\partial\vec{D}}{\partial t}\right) \quad . \qquad (35)$$

Equation (35) is the energy conservation theorem

$$\frac{du}{dt} = \vec{\nabla}\cdot\vec{S} + \frac{\partial u}{\partial t} \quad , \qquad (36)$$

where $\vec{S}$ is the Poynting vector

$$\vec{S} = \vec{E}\times\vec{H} \quad , \tag{37}$$

u is the energy density in the electromagnetic field, and

$$\frac{\partial u}{\partial t} = \vec{H}\cdot \frac{\partial \vec{B}}{\partial t} + \vec{E}\cdot \frac{\partial \vec{D}}{\partial t} \quad . \tag{38}$$

The Fourier representation of the electric field is

$$\vec{E} = \int d^3\vec{k}\; d\omega\; \vec{E}_k e^{i\left(\vec{k}\cdot\vec{r}-\omega t\right)} \tag{39}$$

and

$$\vec{D} = \int d^3\vec{k}\; d\omega\; \vec{D}_k e^{i\left(\vec{k}\cdot\vec{r}-\omega t\right)} \quad . \tag{40}$$

Also, by the plasma constitutive relations

$$D_{ki} = \varepsilon_{ij}(\vec{k},\omega)E_{kj} \quad . \tag{41}$$

Furthermore, taking the Fourier transform of equation (34) produces

$$\vec{B}_k = \frac{\vec{k}\times\vec{E}_k}{\omega} \quad . \tag{42}$$

Also,

$$\vec{H} = \frac{\vec{B}}{\mu_o} = \varepsilon_o c^2 \vec{B} \quad . \tag{43}$$

Next, using equations (39) to (43) and the appropriate Fourier representations in equation (38) gives

$$\frac{\partial u}{\partial t} = \int d^3\vec{k}\ d\omega\ d^3k'\ d\omega'\ \ E_{ki}e^{i(\vec{k}\cdot\vec{r}-\omega t)}(-i\omega')\varepsilon_{ij}(\vec{k}',\omega')E_{k'j}e^{i(\vec{k}'\cdot\vec{r}-\omega' t)} \tag{44}$$

$$+\ \varepsilon_o c^2 \int d^3k\ d\omega\ d^3\vec{k}'\ d\omega'\ \frac{(\vec{k}\times\vec{E}_k)_i}{\omega}\ e^{i(\vec{k}\cdot\vec{r}-\omega t)}(-i\omega')\ \frac{(\vec{k}'\times\vec{E}_{k'})_i}{\omega'}\ e^{i(\vec{k}'\cdot\vec{r}-\omega' t)}\ .$$

By an ordinary vector identity,

$$\begin{aligned}(\vec{k}\times\vec{E}_k)_i(\vec{k}'\times\vec{E}_{k'})_i &= (\vec{k}\times\vec{E}_k)\cdot(\vec{k}'\times\vec{E}_{k'})\\ &= \vec{k}\cdot\vec{k}'\vec{E}_k\cdot\vec{E}_{k'} - \vec{k}\cdot\vec{E}_{k'}\vec{k}'\cdot\vec{E}_k\ .\end{aligned} \tag{45}$$

Substituting equation (45) in equation (44) gives

$$\begin{aligned}\frac{\partial u}{\partial t} = -i \int dk\ dk' \Big[&\omega' E_{ki}\varepsilon_{ij}(\vec{k}',\omega')E_{k'j}\\ &+ \frac{\varepsilon_o c^2}{\omega}\left(\vec{k}\cdot\vec{k}'\vec{E}_k\cdot\vec{E}_{k'} - \vec{k}\cdot\vec{E}_{k'}\vec{k}'\cdot\vec{E}_k\right)\Big]\\ &\times e^{i[(\vec{k}+\vec{k}')\cdot\vec{r}-(\omega+\omega')t]}\ ,\end{aligned} \tag{46}$$

where $dk \equiv d^3\vec{k}\ d\omega$. Symmetrizing equation (46) in k and k' produces

$$\frac{\partial u}{\partial t} = -\frac{i}{2}\int dk\, dk'\left\{E_{k'i}\varepsilon_{ij}(\vec{k},\omega)\omega E_{kj} + E_{ki}\omega'\varepsilon_{ij}(k',\omega')E_{k'j}\right.$$

$$\left. + \varepsilon_o c^2(\omega + \omega')\left[\frac{\vec{k}\cdot\vec{k}'}{\omega\omega'}\vec{E}_k\cdot\vec{E}_{k'} - \frac{(\vec{k}\cdot\vec{E}_{k'})(\vec{k}'\cdot\vec{E}_k)}{\omega\omega'}\right]\right\} \tag{47}$$

$$\times\, e^{i\left[(\vec{k}+\vec{k}')\cdot\vec{r}-(\omega+\omega')t\right]} \quad .$$

The total field involved in the bremsstrahlung process, $\vec{E}_k$, is given elsewhere,[5] namely,

$$\vec{E}_k = \vec{E}_k^{\sigma(0)} + \vec{E}_k^R \quad , \tag{48}$$

where $\vec{E}_k^{\sigma(0)}$ is the lowest order stochastic bremsstrahlung radiation field and $\vec{E}_k^R$ is the regular nonradiative component. The bremsstrahlung field written in terms of its polarization vector $\vec{e}_k^{\sigma}$ is given by

$$\vec{E}_k^{\sigma(0)} = \vec{e}_k^{\sigma} E_k^{\sigma(0)} \quad , \tag{49}$$

where

$$\vec{e}_k^{\sigma}\cdot\vec{e}_k^{\sigma *} = 1 \quad . \tag{50}$$

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[5] *H. E. Brandt, The Total Field in Collective Bremsstrahlung in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1996 (1983).*

The stochastic properties of the bremsstrahlung field are approximated to the needed order by

$$\langle E_{ki}^{\sigma(0)} \rangle = 0 \tag{51}$$

and

$$\langle E_{ki}^{\sigma(0)} E_{k_1 j}^{\sigma(0)} \rangle = e_{ki}^{\sigma} e_{kj}^{\sigma *} \left| E_k^{\sigma(0)} \right|^2 \delta\left(k + k_1\right) \tag{52}$$

(these are eq (49) and (50) of a previous work[5]). Therefore, substituting equation (48) in equation (47), keeping only the lowest order bremsstrahlung field, integrating equation (47) over time, and using equation (51), we find that the ensemble average energy density in the field is given by

$$\langle u \rangle = \langle u \rangle^{\sigma} + \langle u \rangle^{R} \quad , \tag{53}$$

where

$$\begin{aligned} \langle u \rangle^{\sigma} = \int \langle \frac{\partial u}{\partial t}\, dt \rangle = \frac{1}{2} \int dk\, dk' \Bigg\{ \frac{1}{\omega + \omega'} \Big[& \varepsilon_{ij}(\vec{k}, \omega) \omega \langle E_{kj}^{\sigma(0)} E_{k'i}^{\sigma(0)} \rangle \\ & + \omega' \varepsilon_{ij}\left(\vec{k}', \omega'\right) \langle E_{ki}^{\sigma(0)} E_{k'j}^{\sigma(0)} \rangle \Big] + \varepsilon_o c^2 \frac{\vec{k} \cdot \vec{k}'}{\omega\omega'} \langle \vec{E}_k^{\sigma(0)} \cdot \vec{E}_{k'}^{\sigma(0)} \rangle \\ & - \frac{\varepsilon_o c^2 \langle \vec{k} \cdot \vec{E}_{k'}^{\sigma(0)} \vec{k}' \cdot \vec{E}_k^{\sigma(0)} \rangle}{\omega\omega'} \Bigg\} e^{i\left[\left(\vec{k}+\vec{k}'\right) \cdot \vec{r} - (\omega+\omega')t\right]} \end{aligned} \tag{54}$$

is the bremsstrahlung part, and $\langle u \rangle^{R}$ is the regular nonradiative part. Only $\langle u \rangle^{\sigma}$ is needed for the present work.

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[5] *H. E. Brandt, The Total Field in Collective Bremsstrahlung in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-TR-1996 (1983).*

Substituting equation (52) in equation (54) and using the definition of the four-dimensional delta function,

$$\delta(k) = \delta^3(\vec{k})\delta(\omega) = (2\pi)^{-4} \int d^3\vec{r}\ dt\ e^{-i(\vec{k}\cdot\vec{r}-\omega t)} \quad , \tag{55}$$

produces

$$\begin{aligned}\langle u\rangle^\sigma = \frac{1}{2}\int dk\ dk' &\left\{\frac{1}{\omega+\omega'}\left[e^{\sigma *}_{ki}\varepsilon_{ij}(\vec{k},\omega)\omega e^{\sigma}_{kj} + e^{\sigma}_{ki}\omega'\varepsilon_{ij}(\vec{k}',\omega')e^{\sigma *}_{kj}\right]\delta(\omega+\omega')\right. \\ &\left. + \varepsilon_o c^2\left[\frac{\vec{k}\cdot\vec{k}'}{\omega\omega'}\vec{e}^{\sigma}_{k}\cdot\vec{e}^{\sigma *}_{k} - \frac{\vec{k}\cdot\vec{e}^{\sigma *}_{k}\vec{k}'\cdot\vec{e}^{\sigma}_{k}}{\omega\omega'}\right]\delta(\omega+\omega')\right\}\left|E^{\sigma(0)}_{k}\right|^2\delta^3(\vec{k}+\vec{k}') \quad .\end{aligned} \tag{56}$$

In equation (56) we define

$$I(k) = \int dk'\ \frac{\delta(\omega+\omega')}{(\omega+\omega')}\ \delta^3(\vec{k}+\vec{k}')\left[e^{\sigma *}_{ki}\varepsilon_{ij}(\vec{k},\omega)\omega e^{\sigma}_{kj} + e^{\sigma}_{ki}\omega'\varepsilon_{ij}(\vec{k}',\omega')e^{\sigma *}_{kj}\right] \quad . \tag{57}$$

Changing the variable ω' to $\omega'' = \omega + \omega'$ and integrating over $\vec{k}'$, equation (57) becomes

$$I(k) = \int \frac{d\omega''}{\omega''}\ \delta(\omega'')\left[e^{\sigma *}_{ki}\varepsilon_{ij}(\vec{k},\omega)\omega e^{\sigma}_{kj} + (\omega''-\omega)e^{\sigma}_{ki}\varepsilon_{ij}(-\vec{k},\omega''-\omega)e^{\sigma *}_{kj}\right] \quad . \tag{58}$$

Expanding the second term in the integrand of equation (58) in a Taylor series about $\omega'' = 0$, then

$$\begin{aligned}I(k) = \int \frac{d\omega''}{\omega''}\ \delta(\omega'') &\left\{e^{\sigma *}_{ki}\varepsilon_{ij}(\vec{k},\omega)\omega e^{\sigma}_{kj} - \omega e^{\sigma}_{ki}\varepsilon_{ij}(-\vec{k},-\omega)e^{\sigma *}_{kj}\right. \\ &\left. + \omega''\frac{\partial}{\partial\omega''}\left[(\omega''-\omega)e^{\sigma}_{ki}\varepsilon_{ij}(-\vec{k},\omega''-\omega)e^{\sigma *}_{kj}\right]\right\} \quad .\end{aligned} \tag{59}$$

By the reality of the fields, it follows from equation (41) that

$$\varepsilon_{ij}(-\vec{k},-\omega) = \varepsilon^{*}_{ij}(\vec{k},\omega) \quad . \tag{60}$$

Substituting equation (60) in equation (59) and simplifying the second term causes equation (59) to become

$$I(k) = 2i \int \frac{d\omega''}{\omega''} \delta(\omega'')\omega \text{ Im } e^{\sigma *}_{ki}\varepsilon_{ij}(\vec{k},\omega)e^{\sigma}_{kj} + \frac{\partial}{\partial\omega}\left[\omega e^{\sigma}_{ki}\varepsilon_{ij}(-\vec{k},-\omega)e^{\sigma *}_{kj}\right] \quad . \tag{61}$$

The dielectric constant $\varepsilon^{\sigma}(\vec{k},\omega)$ for mode σ is defined in terms of the dielectric permittivity tensor $\varepsilon_{ij}(\vec{k},\omega)$ and the unit electric polarization vectors $\vec{e}^{\sigma}_{k}$ by

$$\varepsilon^{\sigma}(\vec{k},\omega) = e^{\sigma *}_{ki}\varepsilon_{ij}(\vec{k},\omega)e^{\sigma}_{kj} + \varepsilon_o \frac{c^2}{\omega^2}\left(\vec{k}\cdot\vec{e}^{\sigma}_{k}\right)\left(\vec{k}\cdot e^{\sigma *}_{k}\right) \tag{62}$$

(see Other Works by Tsytovich, Selected Bibliography).

Noting that the second term of equation (62) is real, and substituting equation (62) in equation (61), we obtain

$$I(k) = 2i\omega \text{ Im } \varepsilon^{\sigma}(k,\omega) \int \frac{d\omega''}{\omega''} \delta(\omega'') + \frac{\partial}{\partial\omega}\left[\omega e^{\sigma}_{ki}\varepsilon_{ij}(-\vec{k},-\omega)e^{\sigma *}_{kj}\right] \quad . \tag{63}$$

If mode decay or growth are ignorable, then the mode dielectric constant $\varepsilon^{\sigma}(\vec{k},\omega)$ is real and the first term of equation (63) may be dropped. In that case equation (63) becomes

$$I(k) = \frac{\partial}{\partial\omega}\left[\omega e^{\sigma}_{ki}\varepsilon_{ij}(-\vec{k},-\omega)e^{\sigma *}_{kj}\right] \quad . \tag{64}$$

If we next integrate the second term in equation (56) and substitute equations (50), (57), and (64), equation (56) becomes

$$\langle u\rangle^{\sigma} = \frac{1}{2}\int dk \left[\frac{\partial}{\partial\omega}\left(e^{\sigma}_{ki}\omega\varepsilon_{ij}(-\vec{k},-\omega)e^{\sigma *}_{kj}\right) + \frac{\varepsilon_o c^2}{\omega^2}\left(k^2 - \vec{k}\cdot\vec{e}^{\sigma *}_{k}\vec{k}\cdot\vec{e}^{\sigma}_{k}\right)\right]\left|E^{\sigma(0)}_{k}\right|^2 \quad . \tag{65}$$

Changing the variable of integration from k to -k in the first term only, using the reality property of the fields to replace $e^{\sigma}_{-k}E^{\sigma(0)}_{-k}$ by $e^{\sigma *}_{k}E^{\sigma(0)*}_{k}$, substituting equation (62), and simplifying, causes equation (65) to become

$$\langle u\rangle^{\sigma} = \frac{1}{2}\int dk\left[\frac{\partial}{\partial\omega}\left(\omega\varepsilon^{\sigma}(\vec{k},\omega) - \varepsilon_o\frac{c^2}{\omega}\vec{k}\cdot\vec{e}^{\sigma}_{k}\vec{k}\cdot\vec{e}^{\sigma *}_{k}\right)\right.$$

$$\left. + \frac{\varepsilon_o c^2}{\omega^2}\left(k^2 - \vec{k}\cdot\vec{e}^{\sigma}_{k}\vec{k}\cdot\vec{e}^{\sigma *}_{k}\right)\right]\left|E^{\sigma(0)}_{k}\right|^2 \quad . \tag{66}$$

Simplifying equation (66) results in

$$\langle u\rangle^{\sigma} = \frac{1}{2}\int dk\left[\varepsilon^{\sigma}(\vec{k},\omega) + \omega\frac{\partial\varepsilon^{\sigma}(\vec{k},\omega)}{\partial\omega} + \varepsilon_o\frac{k^2}{\omega^2}c^2\right]\left|E^{\sigma(0)}_{k}\right|^2 \quad . \tag{67}$$

The zeroth-order dispersion relation for the bremsstrahlung wave is given by the poles of the photonic Green's function G_{ij} in equation (22) of a previous paper[5] or, equivalently, the zeros of the determinant of G^{-1}_{ij} in equation (18) of that paper.[5] Thus the bremsstrahlung field $E^{\sigma(0)}_{kj}$ must satisfy

$$\left[\frac{1}{\mu_o(\omega + i\delta)^2}\left(k_i k_j - k^2\delta_{ij}\right) + \varepsilon_{ij}\right]E^{\sigma(0)}_{kj} = 0 \quad . \tag{68}$$

Taking the inner product of equation (68) with $e^{*\sigma}_{k_i}$ and using equations (49), (50), and (62) and $\mu_o^{-1} = \varepsilon_o c^2$ produces

$$\frac{\varepsilon_o c^2}{\omega^2}\left(\vec{e}^{\sigma *}_{k}\cdot\vec{k}\vec{e}^{\sigma}_{k}\cdot\vec{k} - k^2\right) + \varepsilon^{\sigma} - \varepsilon_o\frac{c^2}{\omega^2}\left(\vec{k}\cdot\vec{e}^{\sigma}_{k}\right)\left(\vec{k}\cdot\vec{e}^{\sigma *}_{k}\right) = 0 \tag{69}$$

or

$$\frac{k^2c^2}{\omega^2} = \frac{\varepsilon^{\sigma}}{\varepsilon_o} \quad . \tag{70}$$

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[5] *H. E. Brandt, The Total Field in Collective Bremsstrahlung in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories HDL-TR-1996 (1983).*

Substituting equation (70) in equation (67) results in

$$\langle u\rangle^{\sigma} = \frac{1}{2} \int dk \left(2\varepsilon^{\sigma}(\vec{k},\omega) + \omega \frac{\partial\varepsilon^{\sigma}}{\partial\omega}(\vec{k},\omega)\right) \left| E_{k}^{\sigma(0)} \right|^{2} \quad . \tag{71}$$

Equivalently, equation (71) becomes

$$\langle u\rangle^{\sigma} = \frac{1}{2} \int d^{3}\vec{k}\, d\omega\, \frac{1}{\omega} \frac{\partial}{\partial\omega} \left(\varepsilon^{\sigma}(\vec{k},\omega)\omega^{2}\right) \left| E_{k}^{\sigma(0)} \right|^{2} \quad . \tag{72}$$

One can also express the energy density in the field in terms of the photon number density N_{k}^{σ} as follows:

$$\langle u\rangle^{\sigma} = \int \frac{d^{3}\vec{k}}{(2\pi)^{3}} \omega_{k}^{\sigma} N_{k}^{\sigma} \quad , \tag{73}$$

where ω_{k}^{σ} is the frequency of the mode σ as determined by the dispersion relation. Equivalently, equation (73) may be written

$$\langle u\rangle^{\sigma} = \int \frac{d^{3}\vec{k}\, d\omega}{(2\pi)^{3}} \omega N_{k}^{\sigma} \delta\left(\omega - \omega_{k}^{\sigma}\right) \quad . \tag{74}$$

Comparing equations (72) and (74), we find

$$N_{k}^{\sigma} \delta\left(\omega - \omega_{k}^{\sigma}\right) = \frac{4\pi^{3}}{\omega^{2}} \frac{\partial}{\partial\omega} \left(\varepsilon^{\sigma}(\vec{k},\omega)\omega^{2}\right) \left| E_{k}^{\sigma(0)} \right|^{2} \quad . \tag{75}$$

Equation (29) may be equivalently written

$$\vec{F}_\alpha^\sigma = -\int \frac{d^3\vec{k}\, d\omega\, d^3\vec{\kappa}\, d^3\vec{p}_\beta}{(2\pi)^9} \vec{\kappa}(\vec{\kappa} - \vec{k})\cdot(\vec{\nabla}_{p_\beta} f_{\vec{p}_\beta}) V^\sigma_{p_\alpha, \vec{p}_\beta}(\vec{\kappa},\vec{k}) N^\sigma_{\vec{k}}$$

$$\times\ \delta(\omega - \vec{\kappa}\cdot\vec{v}_\alpha + (\vec{\kappa} - \vec{k})\cdot\vec{v}_\beta)\delta(\omega - \omega^\sigma_{\vec{k}}) \quad , \tag{76}$$

where the integral of a delta function over ω has been inserted. Therefore, after substituting equation (75) in equation (76), we finally obtain

$$\vec{F}_\alpha^\sigma = -4\pi^3 \int \frac{d^3\vec{k}\, d\omega\, d^3\vec{\kappa}\, d^3\vec{p}_\beta}{(2\pi)^9} \frac{1}{\omega^2}\frac{\partial}{\partial\omega}\left(\omega^2\epsilon^\sigma(\vec{k},\omega)\right)\vec{\kappa}(\vec{\kappa} - \vec{k})\cdot(\vec{\nabla}_{p_\beta} f_{\vec{p}_\beta})$$

$$\times\ V^\sigma_{\vec{p}_\alpha, \vec{p}_\beta}(\vec{\kappa},\vec{k})\left|E^{\sigma(0)}_{\vec{k}}\right|^2\delta(\omega - \vec{\kappa}\cdot\vec{v}_\alpha + (\vec{\kappa} - \vec{k})\cdot\vec{v}_\beta) \quad . \tag{77}$$

Equation (77) expresses the general form for the force on a particle due to induced bremsstrahlung. The dispersive properties of the beam-plasma system enter explicitly in equation (77) through the dielectric constant $\epsilon^\sigma(\vec{k},\omega)$ and implicitly through the transition probability.

5. CONCLUSION

The general form has been derived for the collective bremsstrahlung recoil force in a nonequilibrium relativistic beam-plasma system, namely,

$$\vec{F}_\alpha^\sigma = -\int \frac{d^3\vec{p}_\beta\, d^3\vec{\kappa}\, d^3\vec{k}}{(2\pi)^9} \vec{\kappa}(\vec{\kappa} - \vec{k})\cdot(\vec{\nabla}_{p_\beta} f_{\vec{p}_\beta}) V^\sigma_{p_\alpha, \vec{p}_\beta}(\vec{\kappa},\vec{k})$$

$$\times\ N^\sigma_{\vec{k}}\delta(\omega^\sigma_{\vec{k}} - \vec{\kappa}\cdot\vec{v}_\alpha + (\vec{\kappa} - \vec{k})\cdot\vec{v}_\beta) \quad . \tag{78}$$

Here $\vec{F}_\alpha^\sigma$ is the force on a particle of species α and velocity $\vec{v}_\alpha$ due to its participation in a bremsstrahlung process in which it interacts with a particle of species β and velocity $\vec{v}_\beta$, resulting in a momentum transfer $\vec{\kappa} - \vec{k}$ to that particle. A bremsstrahlung photon of wave vector $\vec{k}$ and frequency ω_k^σ in mode σ is emitted. The functions $f_{\vec{p}_\beta}$ and $N_{\vec{k}}^\sigma$ are the particle and photon distributions, respectively, and $V^\sigma_{\vec{p}_\alpha, \vec{p}_\beta}$ is the bremsstrahlung transition rate with energy conservation already factored out. The photon distribution function $N_{\vec{k}}^\sigma$ can be expressed in terms of the bremsstrahlung field by equation (75) to obtain an equivalent form given by equation (77). Equation (78) was used by Akopyan and Tsytovich[1,2] to obtain an expression for the collective bremsstrahlung transition rate by means of direct comparison with another expression for the collective bremsstrahlung recoil force, which was determined from the equation of motion for a dynamically polarized test particle undergoing bremsstrahlung.

The present calculation, together with previous work by the author (see Selected Bibliography), is important for ongoing work in calculating collective radiation processes and conditions for the occurrence of radiative instability in relativistic beam-plasma systems.

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